

## Problem Set For Round 3

December 29, 2011

1. Let  $AD$  be perpendicular on  $BC$  in  $\triangle ABC$ . Find the diameter of the circumcircle of  $\triangle ABC$ .
2. Let  $O$  be the center of a circle with radius 5.  $AB$  is a chord with length 8. What is the distance of the chord from the center?
3. Find all  $m$  such that  $\frac{2011}{m} + \frac{m}{2011}$  is a positive integer.
4. Let  $f_n = f_{n-1} + f_{n-2}$  be the Fibonacci series with  $f_0 = 0, f_1 = 1$ . Find

$$f_1 + f_3 + \dots + f_{2011}$$

5. You have 3 types of 20 books, each type has identical books. There are 5 books of the type 1, 6 of the type 7. In how many ways can you arrange them?
6. If  $n$  is a positive integer such that 9 divides  $n^2 + 4$ . Find the remainder when  $n^6$  is divided by 9.
7. There are 8 lines mutually parallel and 10 horizontal lines. Find the number of parallelograms made by the intersections of these lines.
8. Find all primes  $p, q, r$  such that each divides the sum of others.
9. Find all positive integer  $x$  such that

$$x^4 - 4x^3 + 4x^2 = 48$$