

Bangladesh

Olympiad in Informatics

National Contest
27th March, 2010

hosted by

**Ahsanullah University of Engineering and
Technology**

Name:
Student ID:
College/School:

Task 1 Logical Problem

(100 Points)

1. Given 13 coins, it is known that any 12 of them can be divided into two sets of equal weight. Prove that all of them has same weight.
2. Two brother sold a herd of sheep they owned. Each sheep was sold in price same as number of sheep in herd. The money was divided in following manner, Firs older brother took 10 taka then younger brother took 10 taka. At the end, it was younger brother turn and there was less that 10 taka. To make the division just older brother gave his pen to younger brother. Calculate the price of the pen.
3. What is the final digit of the number

$$7^{(7^{(7^{(7^{(7^7)})})})})}$$

Where there is 1001 sevens? What are the final two digits?

4. Two student of school and few student of college took part in a chess tournament. Each student played once against other. The two students of school got 8 points each while each of the college student got same point. If in each match winner gets 1 and loser gets 0 point or both player gets $\frac{1}{2}$ point if the game is a draw the calculate how many college student participated.
5. Show that

$$\frac{1}{\log_2 N} + \frac{1}{\log_3 N} + \dots + \frac{1}{\log_{100} N} = \frac{1}{\log_{100!} N}$$
6. Let x_1, x_2, \dots, x_n are all either 1 or -1. If

$$x_1 x_2 x_3 x_4 + x_2 x_3 x_4 x_5 + \dots + x_{n-3} x_{n-2} x_{n-1} x_n + x_{n-2} x_{n-1} x_n x_1 + x_{n-1} x_n x_1 x_2 + x_n x_1 x_2 x_3 = 0$$
 Then prove n is divisible by 4.
7. You have to pack N squares of same size (side length S) inside a circle of radius R. Each square MUST touch the circile's boundary at EXACTLY two points. Find the maximum possible value of S. Give your answer in terms of R and N.
8. Find the next FOUR terms for the following sequence,
0, 1, 2, 3, 6, 11, 20, 37, 68,

9. Pack three squares of same size (side length S) inside an equilateral triangle of side length T , what is the minimum value of T ? Give your answer in terms of S .

ans: $T_{\text{Min}} = S * (\sqrt{3} + 3/2)$

10. You have two kinds of arithmetic operation- either Double current value, or increment current value. Starting from 1 find the minimum number of operations to meet 2010. Show the sequence.

11. You and your friend are playing a game. There are N sticks. Any player can take away at least 1 stick and at most K sticks ($K < N$). The player who finds no sticks left loses. You start the game. What is the condition of N and K so that you can guarantee that you will win (no matter how well your friend plays)?

Task 2 to 6 Programming Problem (100*5=500 Points)

Task 2- PRODUCT SUM

Given a sequence of numbers a_1, a_2, \dots, a_n , compute the value of the following expression:

$$S = \sum_{i=1}^n \sum_{j=1}^n a_i a_j (-1)^{(i+j)}$$

INPUT

The first line of the file input contains a positive number n. The following line contains the numbers a_1, a_2, \dots, a_n separated by a single space.

CONSTRAINTS

$$1 \leq n \leq 10^5; 1 \leq a_i \leq 10^5$$

For test cases worth 15 points n will be less than 1000.

OUTPUT

The first line of output should contain a single number, the required value.

EXAMPLE INPUT AND OUTPUT

Input	Output
5 1 1 1 1 1	1

PROGRAMMING SUGGESTION

Use **long long** datatype to calculate the answer. The format specifier for **long long** is “%lld”.

Task 3- Round Trip

PROBLEM

Your team decided to take a round trip in the host country after the competition. You want to travel to a destination city and return to the starting city. The only requirement your team specified is that the forward route to the destination city and the return route back to the starting city must contain the least possible number of common roads. (A route can not contain any road twice or more times.)

You are to write a program that computes two routes between the starting city and the destination city so that the number of common roads in the two routes is as small as possible.

INPUT

The first line of the input contains two integers, S and D ($S < D$), the labels of the starting and the destination cities, respectively. The second line contains two integers, N and M , where N ($3 \leq N \leq 1000$) is the number of the cities and M ($2 \leq M \leq 100000$) is the number of the roads between the cities. The cities are labeled from 1 to N . Each of the next M lines in the file contains two integers, P and Q ($1 \leq P, Q \leq N, P < Q$), meaning that there is a two-way road between city P and city Q . There is at most one road between any two cities.

OUTPUT

The first line of the output must contain one integer, the least possible number of common roads of the forward and the return routes. The second line should contain a forward route as a sequence of city labels, including the starting and ending city. The third line should contain a return route as a sequence of city labels (again including the starting and ending cities). If there are more possible pairs of routes with the same least number of common roads then your program may output any one of them. If there is no route from the starting city to the destination city then the first and only line must contain -1.

EXAMPLE INPUT AND OUTPUT

Input	Output
1 6	2
7 8	1 3 2 4 5 7 6
2 1	6 5 4 2 1
1 3	
2 3	
4 2	
4 5	
5 6	

75	
67	

GRADING

If the first line of the output file contains the correct answer then you obtain 40% point on that testcase. If the first line contains a correct solution and the second and third lines contain correct routes then you obtain full score on that testcase.

Task 4- Great Win

Now you and your two best friends have won huge amount of money from a lottery. Now it is time to divide the money among yourself. But unfortunately the amount is not divisible by three. As a programmer you are wondering what is the minimum amount no less than the amount you have won would ensure perfect distribution.

INPUT

The first line of the input contains a positive number n . Representing the amount you have won.

CONSTRAINTS

$$1 \leq n \leq 10^{100}$$

For test cases worth 15 points n will be less than 100000000.

OUTPUT

The first line of output should contain a single number, the required value.

EXAMPLE INPUT AND OUTPUT

Input	Output
5	6

Input	Output
15	15

Task 5- Valid Number

You like some numbers. You say a number >1 is valid if it can be written as multiplication of numbers you like. For example if you like $\{3,4\}$, then valid numbers smaller than 20 are 3,4,9,12 and 16. For this problem given the set of number you like you have to find how many valid numbers are there between two number a,b.

INPUT

The first line of the input contains a positive number p , size of the set of number you like. Next line contains p integers s_i , the numbers you like. Next line contains two number a,b.

CONSTRAINTS

$$1 \leq p \leq 10$$

$$1 < s_i \leq 50$$

$$1 < a \leq b \leq 10^9$$

For test cases worth 15 points p will be less than 4.

For test cases worth another 15 points $(b-a)$ will be less than 100000

OUTPUT

The first line of output should contain a single number, the required value.

EXAMPLE INPUT AND OUTPUT

Input	Output
2 3 4 2 20	5

Input	Output
2 3 4 10 20	2

Task 6- Stone

There are some stones lying on the road. You have to arrange them in non-decreasing order of their weight. You can move any stone to any position. You want to minimize the total weight (sum of the weight of the stones) you have to move.

Input

The first line of the file stone.in contains a positive number n . The following line contains the numbers a_1, a_2, \dots, a_n separated by a single space.

Constraints

$1 \leq n \leq 10^5; 1 \leq a_i \leq 10^4$.

For test cases worth 50 point, n will be less than or equal to 1000.

Output

The first line of stone.out should contain a single integer: minimum total weight you have to move.

Input	Output
4 7 1 2 3	6
4 7 1 2 5	7
11 6 4 5 3 8 2 7 2 11 2 2	24